# LESSON 4.4a

## Special Polynomial Factoring Patterns

**Today you will:**

- Factor polynomials with degree greater than 2.

- Practice using English to describe math processes and equations

**Core Vocabulary:**

- Factored completely, p. 180
    - When a polynomial is written as a product of unfactorable polynomials.
    - When it can’t be factored any more … duh!

# Common factoring patterns we've seen before

**Two squares (difference of)**

$$a^2 - b^2 = (a + b)(a - b)$$

**Perfect squares**

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

**Perfect cubes**

$$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$$

$$a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$$

Factor each polynomial completely.

**a.** $x^3 - 4x^2 - 5x$ **b.** $3y^5 - 48y^3$ **c.** $5z^4 + 30z^3 + 45z^2$

SOLUTION

**a.** $x^3 - 4x^2 - 5x = x(x^2 - 4x - 5)$ Factor common monomial.

$= x(x - 5)(x + 1)$ Factor trinomial.

**b.** $3y^5 - 48y^3 = 3y^3(y^2 - 16)$ Factor common monomial.

$a^2 - b^2$

$= 3y^3(y + 4)(y - 4)$ Difference of Two Squares Pattern

$(a + b)(a - b)$ $a^2 - b^2 = (a + b)(a - b)$

**c.** $5z^4 + 30z^3 + 45z^2 = 5z^2(z^2 + 6z + 9)$ Factor common monomial.

$a^2 + 2ab + b^2$

$= 5z^2(z + 3)^2$ Perfect Square Trinomial Pattern

$(a + b)^2$ $a^2 + 2ab + b^2 = (a + b)^2$

Factor each polynomial completely.

**a.** $x^3 - 7x^2 + 10x$ **b.** $3n^7 - 75n^5$ **c.** $8m^5 - 16m^4 + 8m^3$

SOLUTION

**a.** $x^3 - 7x^2 + 10x = x(x^2 - 7x + 10)$ Factor common monomial.

$= x(x - 5)(x - 2)$ Factor trinomial.

**b.** $3n^7 - 75n^5 = 3n^5(n^2 - 25)$ $a^2 - b^2$ Factor common monomial.

$= 3n^5(n - 5)(n + 5)$ $(a + b)(a - b)$ Difference of Two Squares Pattern

$a^2 - b^2 = (a + b)(a - b)$

**c.** $8m^5 - 16m^4 + 8m^3 = 8m^3(m^2 - 2m + 1)$ $a^2 - 2ab + b^2$ Factor common monomial.

$= 8m^3(m - 1)^2$ $(a - b)^2$ Perfect Square Trinomial Pattern

$a^2 - 2ab + b^2 = (a - b)^2$

# Core Concept

## Special Factoring Patterns

### Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

**Example** $a^3 + b^3$

$$64x^3 + 1 = (4x)^3 + 1^3$$
$$= (4x + 1)(16x^2 - 4x + 1)$$

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### Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

**Example** $a^3 - b^3$

$$27x^3 - 8 = (3x)^3 - 2^3$$
$$= (3x - 2)(9x^2 + 6x + 4)$$

Factor (a) $x^3 - 125$ and (b) $16s^5 + 54s^2$ completely.

SOLUTION

**a.** $x^3 - 125 = \overset{a^3 - b^3}{x^3 - 5^3}$ Write as $a^3 - b^3$.

$= (x - 5)(x^2 + 5x + 25)$ Difference of Two Cubes Pattern

$(a - b)(a^2 + ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

**b.** $16s^5 + 54s^2 = 2s^2(8s^3 + 27)$ Factor common monomial.

$= 2s^2 [(2s)^3 + 3^3]$ Write $8s^3 + 27$ as $a^3 + b^3$.

$= 2s^2(2s + 3)(4s^2 - 6s + 9)$ Sum of Two Cubes Pattern

$(a + b)(a^2 - ab + b^2)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factor (a) $a^3 + 27$ and (b) $6z^5 - 750z^2$ completely.

SOLUTION

**a.** $a^3 + 27 = a^3 + 3^3$ $\quad a^3 + b^3$

Write as $a^3 + b^3$.

$= (a + 3)(a^2 - 3a + 9)$ $\quad (a + b)(a^2 - ab + b^2)$

Sum of Two Cubes Pattern

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

**b.** $6z^5 - 750z^2 = 6z^2(z^3 - 125)$

Factor common monomial.

$= 6z^2 [z^3 - 5^3]$ $\quad a^3 - b^3$

Write $z^3 + 125$ as $a^3 + b^3$.

$= 6z^2(z - 5)(z^2 + 5z + 25)$ $\quad (a - b)(a^2 + ab + b^2)$

Difference of Two Cubes Pattern

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

# Homework

Pg 184, #5-20